MEASUREMENTS

Units

Units are the chosen standards which when applied in front of a number, gives complete information regarding the physical quantity i.e., the reference standard of measurement is known as a unit. Unit can be classified into two categories:

(i) Fundamental or basic units

(ii) Derived units

Fundamental Units : Fundamental units are those units which are independent and not related to each other. They have following properties:

(a) They are well defined and of a suitable size.

(b) They are easily reproduceable at all places.

(c) They do not vary with time and temperature, pressure etc.

These are as follows :

Fundamental SI Units

Physical Quantity	SI Units	
	Name	Symbol
1. Length	metre	m
2. Mass	kilogram	kg
3. Time	second	s
4. Temperature	kelvin	K
5. Electric current	ampere	A
6. Luminous intensity	candela	cd
7. Amount of substance	mole	mol

Derived Units : All those units which can be obtained from seven fundamental units, are called as derived units. Some of the derived units are as follows :

Some Derived Units

Physical quantity	SI Units	
	Name	Symbol
Velocity	–	ms^{-1}
Density	–	$kg\ m^{-3}$
Force	Newton	N
Pressure	Pascal	Pa, Nm^{-2}
Energy, work	Joule	J, Nm

Power	Watt	W, Js^{-1}
Momentum	–	kg ms^{-1}
Surface tension	–	Nm^{-1}
Torque, couple	–	Nm
Moment of inertia	–	kg m^2
Coefficient of viscosity	–	kg $m^{-1}s^{-1}$
Frequency	Hertz	Hz, s^{-1}
Electric charge	Coulomb	C
Electric potential difference	Volt	V
Electric resistance	Ohm	Ω
Electric capacitance	Farad	F
Magnetic flux	Weber	Wb

Definitions of Some Important SI Units :

(i) **Metre (m) :** It is the length equal to 1,650,763.73 wavelengths (in vacuum) of the radiation corresponding to the transition between $2p_{10}$ and $5d_5$ levels of the Kripton atom of mass 86.

(ii) **Kilogram (kg) :** It is the mass of the cylindrical prototype of the kilogram kept at International Bureau of Weights and Measures at Sevres, near Paris.

(iii) **Second (s) :** It is the duration of 9,192,631,770 periods of the radiation

Principal System of Units

S. No.	Name of System	Mass	Length	Time	Current	Temp.	Luminous Intensity	Qty. of matter
1.	F.P.S. System (3 fundamentals)	Pound (lb)	Foot (ft)	Second (sec.)	–	–	–	–
2.	C.G.S. System (3 fundamentals)	Gram (gm)	Centimetre (cm)	Second (sec.)	–	–	–	–
3.	M.K.S. System (3 fundamentals)	Kilogramme (kg)	Metre (m)	Second (sec.)	–	–	–	–
4.	M.K.S.A. System or Georgi System (4 fundamentals)	Kilogram (kg)	Metre (m)	Second (sec.)	Ampere (A)	–	–	–
5.	S.I. System (7 fundamentals) +2 supplementary*	Kilogram (kg)	Metre (m)	Second (sec.)	Ampere (A)	Kelvin (K)	Candella (cd)	Mole (mol)

Supplementary units are *(a)* radian (rad) for Angle (plane angle) *(b)* Steradian (Sr) for Solid Angle.

without any perturbation taking place between the hyperfine levels (F = 4, M = 0 and F = 3, M = 0) of the ground state of Cesium (133) atom.

(iv) **Kelvin (K) :** It is equal to the fraction $\left(\frac{1}{273.16}\right)$ of the thermodynamic temperature of the triple point of water.

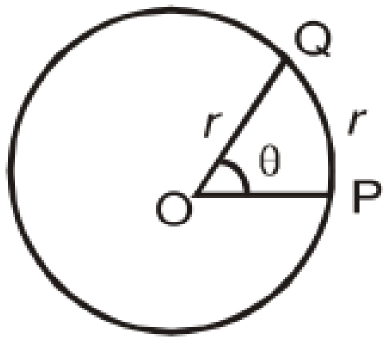
(v) **Ampere (A) :** It is the constant current which produces a force equal to 2×10^{-7} newton per metre length between two straight parallel conductors of infinite length and negligible cross-section, placed one metre apart in vacuum.

(vi) **Candela (cd) :** It is equal to the luminous intensity taken in the perpendicular direction of a surface of $\frac{1}{600,000}$ square metre of a black body at the temperature of freezing platinum under a pressure of 101,325 newton per square metre.

(vii) **Mole (mol) :** It is the amount of substance which contains the same number of elementary entities as there are atoms in 0.012 kg of pure carbon-12 (C^{12}).

(viii) **Radian (rad) :** It is defined as the plane angle subtended at the centre of a circle by an arc of the circle equal in length to its radius, i.e.,

$$\theta \text{ (in radian)} = \frac{\text{arc PQ}}{\text{radius OP}}$$



(ix) **Steradian (Sr) :** It is equal to the solid angle subtended at the centre of a sphere by a surface of the sphere equal in area to that of a square of side equal to the radius of the sphere.

Some more units of length :

$$1 \text{ fermi (f)} = 10^{-15} \text{ m}$$
$$1 \text{ angustrom (Å)} = 10^{-10} \text{ m}$$
$$1 \text{ astronomical (AU)} = 1.49 \times 10^{11} \text{ m}$$
$$1 \text{ light year (ly)} = 9.46 \times 10^{15} \text{ m}$$
$$1 \text{ par sec} = 3.08 \times 10^{16} \text{ m}$$

Inertial mass and gravitational mass :

Inertial mass of an object is that which is defined in terms of its inertial force. It is measured as the ratio of the force acting on the object to the acceleration produced therein, i.e.,

$$m = \frac{F}{a}$$

Gravitational mass of an object is that which is defined in terms of force of gravity acting on it. It is measured as the ratio of the force of gravity acting on the object to the acceleration due to gravity, i.e.,

$$m = \frac{F}{g}$$

Hence, inertial and gravitational masses are equal.

Dimensions : Dimensions of a physical quantity are defined as the powers, which on raising to the fundamental units give the units of that physical quantity.

Dimensional formula : An expression which shows the way of dependence of the unit of a physical quantity on the fundamental units is known as dimensional formula of that physical quantity.

Dimensional equation : An equation consisting of dimensional formulae of various physical quantities is called as dimensional equation.

Principle of homogeneity : According to this principle, the dimensional formulae of all terms on both the sides of the sign of equality of an equation must be exactly the same.

Dimensional Formulae of Some Physical Quantities

Physical quantity	Physical formula	Dimensional formula	Dimensional formula in short form
1. Displacement	difference of two lengths i.e., $(l_2 - l_1)$	$M^0L^1T^0$	L
2. Area	length × breadth	$M^0L^2T^0$	L^2
3. Volume	$l \times b \times h$	$M^0L^3T^0$	L^3
4. Velocity	distance/time	$M^0L^1T^{-1}$	LT^{-1}
5. Acceleration	velocity/time	$M^0L^1T^{-2}$	LT^{-2}
6. Linear momentum	mass × velocity	$M^1L^1T^{-1}$	MLT^{-1}
7. Kinetic energy	$\frac{1}{2}$ mass × (velocity)^2	$M^1L^2T^{-2}$	ML^2T^{-2}
8. Force	mass × acceleration	$M^1L^1T^{-2}$	MLT^{-2}
9. Work	force × distance	$M^1L^2T^{-2}$	ML^2T^{-2}
10. Power	work/time	$M^1L^2T^{-3}$	ML^2T^{-3}
11. Density	mass/volume	$M^1L^{-3}T^0$	ML^{-3}
12. Pressure	force/area	$M^1L^{-1}T^{-2}$	$ML^{-1}T^{-2}$
13. Stress	force/area	$M^1L^{-1}T^{-2}$	$ML^{-1}T^{-2}$
14. Strain	dimensionless	$M^0L^0T^0$	No dimension
15. Modulus of elasticity	stress/strain	$M^1L^{-1}T^{-2}$	$ML^{-1}T^{-2}$
16. Torque (or couple)	force × length of the arm	$M^1L^2T^{-2}$	ML^2T^{-2}

17. Impulse	force × time	$M^1L^1T^{-1}$	MLT^{-1}
18. Surface tension	force/length	$M^1L^0T^{-2}$	MT^{-2}
19. Angle	arc/radius	$M^0L^0T^0$	No dimension
20. Angular velocity	angle/time	$M^0L^0T^{-1}$	T^{-1}
21. Angular Acceleration	$\frac{\text{change in angular velocity}}{\text{time}}$	$M^0L^0T^{-2}$	T^{-2}
22. Frequency	$\frac{\text{Number of vibrations}}{\text{time}}$	$M^0L^0T^{-1}$	T^{-1}
23. Coefficient of viscosity	$\frac{\text{force/area}}{\text{velocity/distance}}$	$M^1L^{-1}T^{-1}$	$ML^{-1}T^{-1}$
24. Gravitational constant (G)	$\frac{\text{force} \times (\text{distance})^2}{\text{mass}(m_1) \times \text{mass}(m_2)}$	$M^{-1}L^3T^{-2}$	$M^{-1}L^3T^{-2}$
25. Thermal conductivity	$\frac{\text{heat energy/distance}}{\text{area} \times \text{temp. diff.} \times \text{time}}$	$M^1L^1T^{-3}K^{-1}$	$MLT^{-3}K^{-1}$
26. Force constant	force/displacement	$M^1L^0T^{-2}$	MT^{-2}

27. Planck's constant	energy × time	$M^1L^2T^{-1}$	ML^2T^{-1}
28. Boltzmann constant	energy/temperature	$M^1L^2T^{-2}K^{-1}$	$ML^2T^{-2}K^{-1}$
29. Latent heat	$\frac{\text{heat energy}}{\text{mass}}$	$M^0L^2T^{-2}$	L^2T^{-2}
30. Specific heat	$\frac{\text{heat energy}}{\text{mass} \times \text{temp.}}$	$M^0L^2T^{-2}K^{-1}$	$L^2T^{-2}K^{-1}$
31. Gravitational potential	work/mass	$M^0L^2T^{-2}$	L^2T^{-2}
32. Universal Gas constant	$R = PV/T$	$M^1L^2T^{-2}\theta^{-1}$	$ML^2T^{-2}\theta^{-1}$
33. Capacitance	$\frac{\text{charge}}{\text{potential difference}}$	$M^1L^{-2}T^4A^2$	$M^{-1}L^{-2}T^4A^2$
34. Resistance	$\frac{\text{potential difference}}{\text{current}}$	$M^1L^2T^{-3}A^{-2}$	$ML^2T^{-3}A^{-2}$
35. Intensity of illumination	$\frac{\text{luminous intensity}}{(\text{distance})^2}$	$M^0L^{-2}T^0Cd^1$	$L^{-2}Cd$

Dimensional equation : The dimensional equation of a physical quantity X is given by

$$X = \left[M^a L^b T^c\right]$$

Use of dimensional equation :

(*a*) To convert one system of units into another system.

(*b*) To check the correctness of any given physical relation.

(*c*) To drive a physical relationship between various physical quantities.

Following formula is used :

$$n_1 = n_2\left[\frac{M_2}{M_1}\right]^a \cdot \left[\frac{L_2}{L_1}\right]^b \cdot \left[\frac{T_2}{T_1}\right]^c$$

Limitation of dimensional analysis :

(*i*) The method does not tell us anything about dimensionless constants and numerical numbers which may appear in physical expressions. They have to be determined by experimental methods.

(*ii*) It does not explain whether the quantity under consideration is a vector or scalar.

(*iii*) It is also found useless in case of trigonometric, logarithmic, exponential and complex quantities.

(iv) Because only three independent equations can be obtained by equating the dimensions of M, L and T, so this method appears of no avail in finding the exact form of a physical relation which depends upon more than three physical quantities.

(v) The method also fails to derive a physical relation which has more than one term on either side of the sign of equality.

Principle of Homogeneity : The dimensions of both sides in an equation are same. For example,

$$S = ut + \frac{1}{2}at^2$$

$$[L] = [LT^{-1}.T] + [LT^{-2}.T^2]$$

$$[L][T^0] = [L] + [L]$$

Errors in Measurements : There is a limit of accuracy to the measuring instrument as well as to the person who is measuring the physical quantity. This means that the exact measurement of a physical quantity is not possible. This is Heisenberg's uncertainty principle. This uncertainty in the measurements is called the error.

If there are several measurements: $x_1, x_2, x_3, ..., x_n$, then the best possible value of the measured quantity is given by,

$$x_{\text{mean}} = \frac{x_1 + x_2 + x_3 + ... + x_n}{n}$$

$$\text{or } \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Absolute error : It is defined as the magnitude of the difference between the value of the quantity under consideration and its individual observation.

Mathematically,

$$\Delta\bar{x} = \frac{|\Delta x_1| + |\Delta x_2| + |\Delta x_3| + ... |\Delta x_n|}{n}$$

$$= \frac{1}{n}\sum_{i=1}^{n} |\Delta x_i|$$

Relative error : It is defined as the ratio of the mean absolute error to the value of the quantity under measurement. i.e.,

$$\delta x = \frac{\Delta\bar{x}}{\bar{x}}$$

Percentage error :

$$\text{Percentage error} = \frac{\Delta\bar{x}}{\bar{x}} \times 100$$